Nonlinear transport and oscillating magnetoresistance in double quantum wells

N. C. Mamani,[1] G. M. Gusev,[1] O. E. Raichev,[1,2] T. E. Lamas,[1] and A. K. Bakarov[1,3]
[1]*Instituto de Física da Universidade de São Paulo, CP 66318, São Paulo CEP 05315-970, SP, Brazil*
[2]*Institute of Semiconductor Physics, NAS of Ukraine, Prospekt Nauki 45, 03028 Kiev, Ukraine*
[3]*Institute of Semiconductor Physics, Russian Academy of Sciences, Novosibirsk 630090, Russia*
(Received 30 July 2008; revised manuscript received 27 July 2009; published 13 August 2009)

The nonlinear regime of low-temperature magnetoresistance of double quantum wells in the region of magnetic fields below 1 T is studied both experimentally and theoretically. The observed inversion of the magnetointersubband oscillation peaks with increasing electric current and splitting of these peaks are described by the theory based on the kinetic equation for the isotropic nonequilibrium part of electron distribution function. The inelastic-scattering time of electrons is determined from the current dependence of the inversion field.

DOI: 10.1103/PhysRevB.80.075308 PACS number(s): 73.23.−b, 73.50.Fq

I. INTRODUCTION

The nonlinear transport in two-dimensional (2D) electron systems placed in a perpendicular magnetic field has been extensively studied in the past in connection with the breakdown of the quantum Hall effect at high current densities.[1] More recently, it was realized that the resistivity is substantially modified by the electric current in the region of weak magnetic fields and relatively high temperatures, when the Landau levels are thermally mixed, so the quantum Hall effect does not occur and even the Shubnikov-de Haas oscillations (SdHOs) are suppressed. The basic reason for high sensitivity of the resistance to the current is the Landau quantization, which becomes essential in high-mobility systems at the magnetic fields on the order of 0.1 T. In such systems, an electric current generates a substantial Hall field perpendicular to the current flow. In the presence of this field, scattering-assisted transitions of electrons between different Landau levels become possible, which leads to modifications of the resistivity.

The present interest in the static (dc) nonlinear transport in 2D systems is stimulated by the observation of two important phenomena. First, there appear oscillations of the resistance as a function of either magnetic field or electric current.[2–4] Second, the current substantially reduces the resistance even at moderate applied voltages.[3,5,6] The oscillating behavior is a consequence of the geometric resonance in the electron transitions between the tilted Landau levels when the diameter of the cyclotron orbit becomes commensurable with the spatial modulation of the density of states.[2,3,7] The decrease in the resistance is governed by modification of electron diffusion in the energy space, which leads to the oscillating nonequilibrium contribution to the distribution function of electrons.[8] A theory describing both these phenomena in a unified way[9,10] shows that the existence of the oscillations requires the presence of a short-range scattering potential to enable efficient backscattering. The decrease in the resistance, in contrast, occurs for an arbitrary scattering potential. Experimental investigations of this phenomenon[5,6] strongly support the theory[8–10] predicting nontrivial changes in the distribution function as a result of dc excitation under magnetic fields. Nevertheless, further studies are necessary for better understanding of the physical mechanisms of this nonlinear behavior.

In this paper, we investigate nonlinear magnetotransport in double quantum wells (DQWs), which are representative for the systems with two closely spaced occupied 2D subbands. In contrast to the quantum wells with a single occupied subband, the positive magnetoresistance, which appears owing to the Landau quantization,[11] is modulated in DQWs by the magnetointersubband (MIS) oscillations.[12] These oscillations, whose maxima correspond to integer ratios of the subband splitting energy Δ_{12} to the cyclotron energy $\hbar\omega_c$, are caused by periodic variation in the probability of elastic intersubband scattering of electrons by the magnetic field as the density of electron states oscillates with energy. Owing to this property, the changes in the quantum contribution to the resistivity are directly seen from the corresponding changes in the MIS oscillation amplitudes. In particular, we observe a remarkable manifestation of nonlinearity in DQWs, the inversion of the MIS oscillation picture, which appears when the quantum magnetoresistance changes from positive to negative with increasing current (Fig. 1). In addition, we observe a splitting of the MIS oscillation peaks in the region

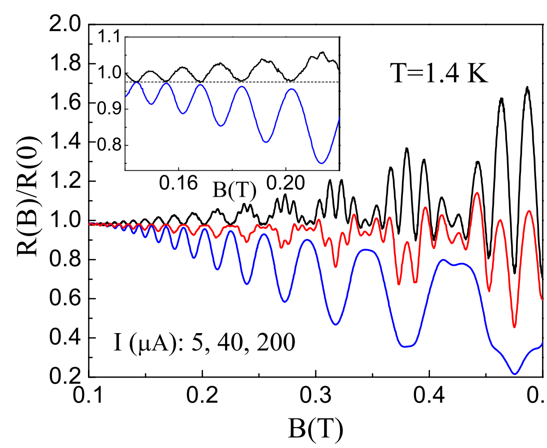

FIG. 1. (Color online) Magnetoresistance of sample A for three different currents I at $T=1.4$ K. The oscillations are inverted with the increase in the current. The inset shows the linear and the nonlinear (at $I=200$ μA) magnetoresistance in the low-field region.

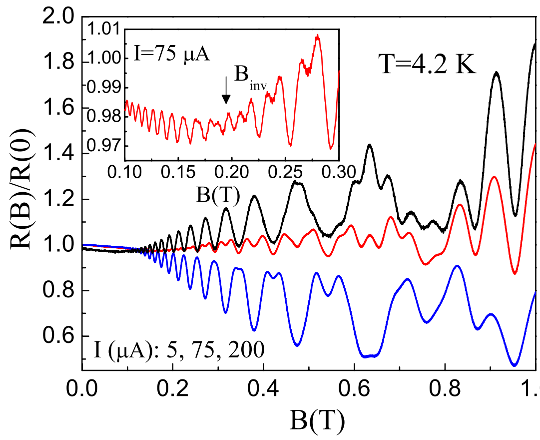

FIG. 2. (Color online) Magnetoresistance of sample A for different currents at $T=4.2$ K. The inset shows inversion of the quantum magnetoresistance around $B=0.2$ T.

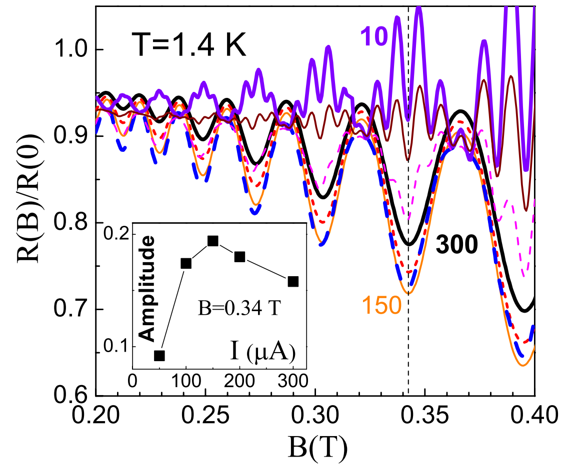

FIG. 3. (Color online) Magnetoresistance of sample B at $T=1.4$ K. The values of the current are 10 (bold curve), 30, 50 (dashed curve), 100 (bold dashed curve), 150, 200 (short dashed curve), and 300 (bold curve) μA. The inset shows amplitudes of the inverted peaks at $B=0.34$ T.

of fields above the inversion point. By adopting basic ideas of the theory,[8] we explain our experimental data and determine the inelastic relaxation time of electrons in our samples.

The paper is organized as follows. In Sec. II we describe the experimental details and present the results of our measurements. In Sec. III we generalize the theory of Ref. 8 to the case of two-subband occupation. A discussion, including a comparison of experimental results with the results of our calculations, is given in Sec. IV. The last section contains the concluding remarks.

II. EXPERIMENT

The samples are symmetrically doped GaAs double quantum wells with equal widths $d_W=14$ nm separated by $Al_xGa_{1-x}As$ barriers with widths $d_b=1.4$, 2, and 3.1 nm. Both layers are shunted by Ohmic contacts. Over a dozen specimens of both the Hall bars and the van der Pauw geometries from three wafers have been investigated. We have studied the dependence of the resistance of symmetric balanced GaAs DQWs on the magnetic field B at different applied voltages and temperatures. While similar results have been obtained for all samples with different configurations and barrier widths, we focus on measurements performed on two samples with barrier width $d_b=1.4$ nm. The samples are Hall bars of width 200 μm and length 500 μm between the voltage probes. The mobilities are 9.75×10^5 cm^2/V s (sample A) and 4.0×10^5 cm^2/V s (sample B), and the total electron density $n_s=1.01\times10^{12}$ cm^{-2} is the same for both samples. The resistance $R=R_{xx}$ was measured by using the standard low-frequency lock-in technique for a low value of the current. We also use dc current, especially for high-current measurements. The results obtained with ac and dc techniques are similar. The subband separation Δ_{12}, found from the MIS oscillation periodicity at low B, is 3.7 meV for sample A and 5.1 meV for sample B.

The resistance of sample A as a function of magnetic field at different temperatures and currents is presented in Figs. 1 and 2. At weak currents, the magnetoresistance is positive and modulated by the large-period MIS oscillations clearly visible above $B=0.1$ T. The small-period SdHOs, superimposed on the MIS oscillation pattern, appear at higher fields in the low-temperature measurements (Fig. 1). With increasing current I, the amplitudes of the MIS oscillations decrease, until a flip of the MIS oscillation picture occurs. This flip, associated with inversion of the quantum part of the magnetoresistance from positive to negative, starts from the region of lower fields and extends to higher fields as the current increases. Therefore, one can introduce a current-dependent inversion field B_{inv}. The inset to Fig. 2 shows the behavior of the magnetoresistance near the point of inversion, where we observe an additional feature that looks like splitting of the MIS oscillation peaks or appearance of the next harmonics of the MIS oscillations. This feature persists in higher magnetic fields. In contrast to the MIS oscillations, the SdHOs are not inverted. However, owing to electron heating by the current, the SdHO amplitudes decrease as the current increases until the SdHOs completely disappear in the low-field region.

The amplitudes of inverted MIS oscillations increase with increasing current and become larger than the MIS oscillation amplitudes in the linear regime. At low temperatures the ratio of the corresponding amplitudes varies between 2 and 3; see Fig. 1. However, when the current increases further, the amplitudes of inverted peaks slowly decrease; this decrease is faster in the region of lower magnetic fields. This property is seen in Figs. 3 and 4, where the magnetoresistance data for sample B are presented. The typical current dependence of the inverted peak amplitudes at $T=1.4$ K is shown in the inset to Fig. 3. The behavior of magnetoresistance at 4.2 K, shown in Fig. 4, is similar. In the chosen interval of magnetic fields, the SdHOs at 4.2 K are suppressed even in the linear regime. The splitting of the MIS oscillation peaks is clearly visible in Fig. 4 at $I=80$ μA. For $I=100$ μA this splitting apparently develops in the frequency doubling of the MIS oscillations. Further increase in the current suppresses this feature, leading to a simpler picture of inverted MIS oscillations.

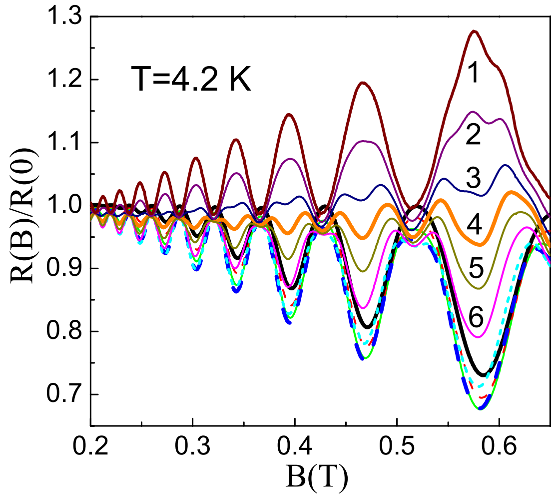

FIG. 4. (Color online) Magnetoresistance of sample B at $T = 4.2$ K. The values of the current (μA) are 1, 50, 80, 100, 120, and 150 for the curves marked by the numbers from 1 to 6, respectively. The other curves correspond to the currents of 200 (short dashed curve), 250 (bold dashed curve) 300 (solid curve), 350 (dashed curve), and 400 (bold curve) μA.

III. THEORY

The theoretical interpretation of our data is based on the physical model of Dmitriev *et al.*,[8] generalized in this section to the two-subband case. According to the theory, the main influence of the current on the dissipative conductivity σ_d is associated with changes in the isotropic part of electron distribution function f_ε, whose first derivative enters the expression for conductivity,

$$\sigma_d = \int d\varepsilon \left(-\frac{\partial f_\varepsilon}{\partial \varepsilon} \right) \sigma_d(\varepsilon). \tag{1}$$

The quantity $\sigma_d(\varepsilon)$ describes the contribution from electrons with a fixed energy ε and is proportional to the squared density of electron states. When the current flows through the sample, it effectively causes a diffusion of electrons in the energy space. This is reflected by the kinetic equation

$$\frac{P}{D_\varepsilon \sigma_d} \frac{\partial}{\partial \varepsilon} \sigma_d(\varepsilon) \frac{\partial}{\partial \varepsilon} f_\varepsilon = - J_\varepsilon(f), \tag{2}$$

where $P = j^2 \rho_d$ is the power of Joule heating (the energy absorbed per unit time over a unit square of electron system), j is the current density, ρ_d is the resistivity, D_ε is the density of electron states, and J_ε is the collision integral. The spectral diffusion is strongest near the centers of the Landau levels, where the density of states, and so $\sigma_d(\varepsilon)$, is maximal and leads to a "flattening" of the distribution function there.[6,8] This means that $-\partial f_\varepsilon / \partial \varepsilon$ becomes smaller compared to the case of equilibrium Fermi distribution in the regions of maximal density of states. Since these regions give the main contribution to conductivity (1), the latter decreases with increasing current. In the classically strong magnetic fields, the resistivity is proportional to the conductivity and behaves in the same way.

Having described the basic idea, in the rest of this section, we give the details of the theoretical formalism in the application to the two-subband systems. First of all, to ensure that the nonequilibrium distribution function f_ε is subband independent (common for both subbands), we assume that the intersubband elastic scattering is much stronger than the inelastic scattering. In symmetric (balanced) DQWs, where the probability of intersubband scattering is close to that of intrasubband scattering, this requirement is easily satisfied. To find D_ε and $\sigma_d(\varepsilon)$ in the magnetic field B, we choose the vector potential as $(0, Bx, 0)$ and describe the free-electron states by the quantum numbers j, n, and p_y, where $j = 1, 2$ numbers the electron subband of the quantum well, n is the Landau-level number, and p_y is the continuous momentum. In this basis,

$$\sigma_d(\varepsilon) = \frac{e^2}{2\pi m} \mathrm{Re}[Q_\varepsilon^{AR} - Q_\varepsilon^{AA}], \tag{3}$$

$$Q_\varepsilon^{ss'} = \frac{2\omega_c}{L^2} \sum_{nn'} \sum_{jj'} \sqrt{(n+1)(n'+1)} \sum_{p_y p_y'} \times \langle\langle G_\varepsilon^{jj',s}(n+1 p_y, n'+1 p_y') G_\varepsilon^{j'j,s'}(n' p_y', n p_y) \rangle\rangle, \tag{4}$$

where e is the electron charge, m is the effective mass of electron, $G_\varepsilon^{jj',s}$ are the retarded ($s = R$) and the advanced ($s = A$) Green's functions, and L^2 is the normalization square. The Zeeman splitting is neglected, so the electrons are assumed to be spin degenerate. The double angular brackets in Eq. (4) denote averaging over the random potential. In terms of the Green's functions, the density of states is given by

$$D_\varepsilon = \frac{2}{\pi L^2} \sum_{jnp_y} \mathrm{Im} \langle\langle G_\varepsilon^{jj,A}(np_y, np_y) \rangle\rangle = \frac{2m}{\pi \hbar^2} \sum_j \mathrm{Im}\, S_{j\varepsilon}. \tag{5}$$

The dimensionless function $S_{j\varepsilon}$ is found from the implicit equation

$$S_{j\varepsilon} = \frac{\hbar\omega_c}{2\pi} \sum_n \frac{1}{\varepsilon - \hbar\omega_c(n+1/2) - \varepsilon_j - \Sigma_{j\varepsilon}},$$

$$\Sigma_{j\varepsilon} = \sum_{j'} \frac{\hbar}{\tau_{jj'}} S_{j'\varepsilon}, \tag{6}$$

where ω_c is the cyclotron energy, ε_j is the subband energy, and $\tau_{jj'}$ are the quantum lifetimes of electrons with respect to intrasubband $(j' = j)$ and intersubband $(j' \neq j)$ scatterings. Equation (6) is valid when the correlation length of the disorder potential is smaller than the magnetic length, and the disorder-induced energy broadening of the subbands is smaller than the subband separation $\Delta_{12} = \varepsilon_2 - \varepsilon_1$. It corresponds to the self-consistent Born approximation (SCBA).

Below, we consider the case of a classically strong magnetic field, $\omega_c \tau_{tr} \gg 1$, when $\sigma_d(\varepsilon)$ is written in terms of $S_{j\varepsilon}$ as

$$\sigma_d(\varepsilon) = \frac{4e^2}{m\omega_c^2}\left[\frac{n_1}{\tau_{11}^{tr}}(\text{Im } S_{1\varepsilon})^2 + \frac{n_2}{\tau_{22}^{tr}}(\text{Im } S_{2\varepsilon})^2 + \frac{n_s}{\tau_{12}^{tr}}\text{Im } S_{1\varepsilon}\,\text{Im } S_{2\varepsilon}\right], \tag{7}$$

where n_1 and n_2 are the electron densities in the subbands, $n_s=n_1+n_2$, and $\tau_{jj'}^{tr}$ are the transport times of electrons. Both $\tau_{jj'}$ and $\tau_{jj'}^{tr}$ are determined by the expressions

$$\left.\begin{array}{l} 1/\tau_{jj'} \\ 1/\tau_{jj'}^{tr} \end{array}\right\} = \frac{m}{\hbar^3}\int_0^{2\pi}\frac{d\theta}{2\pi}w_{jj'}\left[\sqrt{(k_j^2+k_{j'}^2)F_{jj'}(\theta)}\right] \times \left\{\begin{array}{l} 1 \\ F_{jj'}(\theta) \end{array}\right. , \tag{8}$$

where $w_{jj'}(q)$ are the Fourier transforms of the correlators of the scattering potential, $F_{jj'}(\theta)=1-2k_jk_{j'}\cos\theta/(k_j^2+k_{j'}^2)$, and k_j is the Fermi wave number for the subband j. The electron densities in the subbands are expressed as $n_j=k_j^2/2\pi$.

In DQWs, where the energy separation between the subbands is usually small compared to the Fermi energy, the difference $k_1^2-k_2^2$ is small in comparison with $k_1^2+k_2^2$, so that $n_1 \simeq n_2 \simeq n_s/2$. Furthermore, in the balanced DQWs, where the electron wave functions are delocalized over the layers and represent themselves symmetric and antisymmetric combinations of single-layer orbitals, one has nearly equal probabilities for intrasubband and intersubband scatterings owing to $w_{11}(q) \simeq w_{22}(q) \simeq w_{12}(q)$, provided that interlayer correlation of the scattering potentials is weak. Therefore, $\tau_{jj} \simeq \tau_{12} \simeq 2\tau$ and $\tau_{jj}^{tr} \simeq \tau_{12}^{tr} \simeq 2\tau_{tr}$, where τ and τ_{tr} are the averaged quantum lifetime and the transport time, respectively. In these approximations, Eq. (7) is written in the simplest way as follows:

$$\sigma_d(\varepsilon) \simeq \sigma_d^{(0)}\mathcal{D}_\varepsilon^2, \quad \mathcal{D}_\varepsilon = \tfrac{1}{2}(\mathcal{D}_{1\varepsilon}+\mathcal{D}_{2\varepsilon}), \quad \mathcal{D}_{j\varepsilon} = 2\text{ Im } S_{j\varepsilon}, \tag{9}$$

where $\sigma_d^{(0)}=\sigma_\perp^2\rho_0$, $\sigma_\perp=e^2n_s/m\omega_c$ is the Hall conductivity, and $\rho_0=m/e^2\tau_{tr}n_s$ is the classical resistivity.

The function $\mathcal{D}_\varepsilon=1+\gamma_\varepsilon$ is the dimensionless (i.e., normalized to its zero-field value) density of states, containing an oscillating (periodic in $\hbar\omega_c$) part γ_ε. Therefore, it is convenient to solve the kinetic equation by representing the distribution function as a sum $f_\varepsilon^0+\delta f_\varepsilon$, where the first term slowly varies on the scale of cyclotron energy, while the second one rapidly oscillates.[8] The first term satisfies the equation

$$\kappa\frac{\partial^2}{\partial\varepsilon^2}f_\varepsilon^0 = -J_\varepsilon(f^0), \quad \kappa = \frac{\pi\hbar^2 j^2\rho_0}{2m}. \tag{10}$$

The solution of this equation can be satisfactory approximated by a heated Fermi distribution. This is always true if the electron-electron scattering dominates over the electron-phonon scattering and over the electric-field effect described by the left-hand side of Eq. (10). In this case, the Fermi distribution of electrons is maintained against the field-induced diffusion in the energy space, while the electron-phonon scattering determines the effective electron temperature T_e. In the general case, a numerical solution of Eq. (10) involving electron-phonon scattering in the collision integral[13] confirms that f_ε^0 is very close to the heated Fermi distribution. The electron temperature can be found from the balance equation $P=P_{ph}$, where $P_{ph}=-\int d\varepsilon\, \varepsilon D_\varepsilon J_\varepsilon(f)$ is the power lost to the lattice vibrations (phonons). The balance equation is obtained by multiplying the kinetic equation (2) by the density of states D_ε and energy ε and integrating it over ε.

The equation for the oscillating part δf_ε is then written as

$$\mathcal{D}_\varepsilon\frac{\partial^2}{\partial\varepsilon^2}\delta f_\varepsilon + 2\frac{\partial\mathcal{D}_\varepsilon}{\partial\varepsilon}\frac{\partial}{\partial\varepsilon}\delta f_\varepsilon + \kappa^{-1}J_\varepsilon(\delta f) = -2\frac{\partial\mathcal{D}_\varepsilon}{\partial\varepsilon}\frac{\partial f_\varepsilon^0}{\partial\varepsilon}. \tag{11}$$

Below, we search for the function δf_ε in the form $\delta f_\varepsilon=(\partial f_\varepsilon^0/\partial\varepsilon)\varphi_\varepsilon$, where φ_ε is a periodic function of energy. Taking into account that the main mechanism of relaxation of the distribution δf_ε is the electron-electron scattering, one may represent the linearized collision integral $J_\varepsilon(\delta f)$ as

$$J_\varepsilon(\delta f) = -\frac{1}{\tau_{in}}\frac{\partial f_\varepsilon^0}{\partial\varepsilon}\frac{1}{\mathcal{N}\mathcal{D}_\varepsilon}\sum_{jj'j_1j_1'}M_{jj',j_1j_1'}\langle\mathcal{D}_{j\varepsilon}\mathcal{D}_{j_1\varepsilon+\delta\varepsilon}\mathcal{D}_{j'\varepsilon'}\mathcal{D}_{j_1'\varepsilon'-\delta\varepsilon}$$
$$\times[\varphi_\varepsilon+\varphi_{\varepsilon'}-\varphi_{\varepsilon+\delta\varepsilon}-\varphi_{\varepsilon'-\delta\varepsilon}]\rangle_{\varepsilon',\delta\varepsilon},$$
$$\mathcal{N} = \sum_{jj'j_1j_1'}M_{jj',j_1j_1'}, \tag{12}$$

where $\delta\varepsilon$ is the energy transferred in electron-electron collisions, $M_{jj',j_1j_1'}$ is the probability of scattering (when electrons from the states j and j' come to the states j_1 and j_1'), $\mathcal{N}$ is the normalization constant, and the angular brackets $\langle\cdots\rangle_{\varepsilon',\delta\varepsilon}$ denote averaging over the energies ε' and $\delta\varepsilon$. Expression (12) is a straightforward generalization of the result of Ref. 8. The characteristic inelastic-scattering time τ_{in} describes the relaxation at low magnetic fields, when $\mathcal{D}_{j\varepsilon}$ are close to unity. In this case the collision integral acquires the simplest form $J_\varepsilon(\delta f)=-\delta f_\varepsilon/\tau_{in}$, i.e., the relaxation time approximation is justified.

The resistivity $\rho_d=\sigma_d^{(0)}/\sigma_\perp^2$ is written, according to Eq. (1), in the form

$$\rho_d = \rho_0\int d\varepsilon\,\mathcal{D}_\varepsilon^2\left(-\frac{\partial f_\varepsilon^0}{\partial\varepsilon}\right)\left(1+\frac{\partial\varphi_\varepsilon}{\partial\varepsilon}\right), \tag{13}$$

where we have taken into account that $\partial f_\varepsilon/\partial\varepsilon \simeq (\partial f_\varepsilon^0/\partial\varepsilon)[1+\partial\varphi_\varepsilon/\partial\varepsilon]$. Therefore, in order to calculate the resistivity, one should find φ_ε by using Eqs. (11) and (12). In general, Eq. (11) is an integrodifferential equation that cannot be solved analytically. However, the property of periodicity allows one to expand φ_ε in series of harmonics, $\varphi_\varepsilon=\sum_k\varphi_k\exp(2\pi ik\varepsilon/\hbar\omega_c)$, and represent Eq. (11) as a system of linear equations

$$(Q^{-1}+k^2)\varphi_k + \sum_{k'=-\infty}^{\infty}[(2kk'-k'^2)\gamma_{k-k'}+Q^{-1}C_{kk'}]\varphi_{k'} = 2ik\frac{\hbar\omega_c}{2\pi}\gamma_k, \tag{14}$$

where

$$Q = \frac{2\pi^3 j^2}{e^2 n_s \omega_c^2} \frac{\tau_{in}}{\tau_{tr}} \quad (15)$$

is a dimensionless parameter characterizing the nonlinear effect of the current on the transport. The matrix $C_{kk'}$, whose explicit form is not shown here, describes the effects of electron-electron scattering beyond the relaxation time approximation.

The harmonics of the density of states, γ_k, as well as the coefficients $C_{kk'}$, which are expressed in terms of products of these harmonics, are proportional to the Dingle factors $\exp(-k\pi/\omega_c\tau)$. Therefore, searching for the coefficients φ_k at weak enough magnetic fields, when $e^{-\pi/\omega_c\tau}$ is small, one can take into account only a single ($k=\pm 1$) harmonic. Within this accuracy, one should also neglect the sum in Eq. (14). This leads to a simple solution $\varphi_{\pm 1} = \pm i\gamma_{\pm 1}(\hbar\omega_c/\pi)Q/(1+Q)$. Since $\gamma_{+1}+\gamma_{-1} = -2e^{-\pi/\omega_c\tau}\cos(\pi\Delta_{12}/\hbar\omega_c)$, Eq. (13) is reduced to a simple analytical expression for the resistivity

$$\frac{\rho_d}{\rho_0} = 1 + e^{-2\pi/\omega_c\tau}\frac{1-3Q}{1+Q}\left(1+\cos\frac{2\pi\Delta_{12}}{\hbar\omega_c}\right) - 4e^{-\pi/\omega_c\tau}\mathcal{T}\cos\left(\frac{2\pi\varepsilon_F}{\hbar\omega_c}\right)\cos\left(\frac{\pi\Delta_{12}}{\hbar\omega_c}\right). \quad (16)$$

The second term in this expression, which is proportional to $e^{-2\pi/\omega_c\tau}$, differs from a similar term of the single-subband theory[8] by the modulation factor $[1+\cos(2\pi\Delta_{12}/\hbar\omega_c)]/2$ describing the MIS oscillations. The last term in Eq. (16) describes the SdHOs, which are thermally suppressed because of the factor $\mathcal{T}=(2\pi^2 T_e/\hbar\omega_c)/\sinh(2\pi^2 T_e/\hbar\omega_c)$. The Fermi energy ε_F is counted from the middle point between the subbands, $(\varepsilon_1+\varepsilon_2)/2$, and, therefore, is directly proportional to the total electron density, $\varepsilon_F = \hbar^2\pi n_s/2m$.

IV. RESULTS AND DISCUSSION

A. MIS peak inversion and inelastic-scattering time

The basic features of our experimental findings can be understood within Eq. (16). In the linear regime, when the parameter Q is small, this equation gives a good description of the MIS oscillations experimentally investigated in Ref. 12. As the current increases, the amplitudes of the MIS peaks decrease until the peak flip occurs. Then, the MIS oscillations become inverted and their amplitude grows again. In contrast, the SdHO peaks are not affected by the current directly, and their decrease is caused by the effect of heating. The flip of the MIS oscillations corresponds to $Q=1/3$. Since Q is inversely proportional to the square of the magnetic field, there exists the inversion field B_{inv} determined from the equation $Q=1/3$, where Q is given by Eq. (15). This feature is observed in our experiment (see the inset to Fig. 2). For sample B, we have extracted B_{inv} for several values of the current. The results are shown in Fig. 5. At 4.2 K the experimental points follow the linear $B_{inv}(I)$ dependence predicted by Eq. (15). The only unknown parameter in the right-hand side of Eq. (15) is the inelastic relaxation time τ_{in}. Since the ratio B_{inv}/I is proportional to the square root of τ_{in}, we are able to estimate this time from experimental data

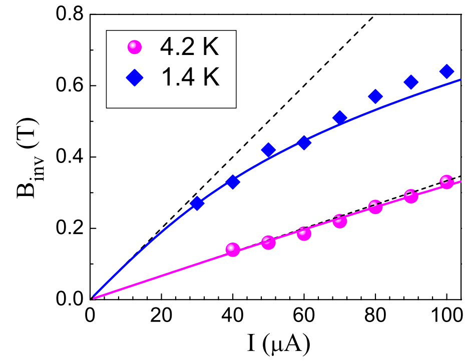

FIG. 5. (Color online) Dependence of the inversion field on the current for sample B at $T=4.2$ K and $T=1.4$ K (points). The dashed lines correspond to a linear $B_{inv}(I)$ dependence assuming $\tau_{in}=64$ ps at 4.2 K (580 ps at 1.4 K). The solid lines represent the calculated $B_{inv}(I)$ dependence taking into account electron heating by the current.

as $\tau_{in}\simeq 64$ ps at $T=4.2$ K. Assuming the T^{-2} scaling of this time,[8] one obtains $\hbar/\tau_{in}=6.6$ mK at $T=1$ K, which is not far than the theoretical estimate $\hbar/\tau_{in}=4$ mK at $T=1$ K based on the consideration of electron-electron scattering.[8] The positions of experimental points at $T=1.4$ K also fit within this picture if the electron heating is taken into account. The increase in electron temperature with increasing current (this heating effect is calculated here by using the collision integral for interaction of electrons with acoustic phonons[13,14]) leads to a deviation of the $B_{inv}(I)$ dependence from the linearity because of temperature dependence of τ_{in}. This deviation is essential at $T=1.4$ K; see Fig. 5. The same consideration, applied to the high-mobility sample A, gives the inelastic-scattering time $\tau_{in}\simeq 108$ ps at $T=4.2$ K, which is very close to the theoretical estimate.

When the current becomes high enough ($Q\gg 1$), Eq. (16) predicts saturation of the resistance, when the amplitudes of inverted MIS peaks are three times larger than the amplitudes of the MIS peaks in the linear regime ($Q\ll 1$). We indeed observe the regime resembling a saturation, with almost three times increase in the amplitudes of inverted peaks for both samples at $T=1.4$ K (see Figs. 1 and 3). For higher temperatures, the behavior is similar, although the maximum amplitudes of inverted peaks are only slightly larger than the amplitudes in the linear regime. We explain this by the effect of heating on the characteristic times. Although the resistivity in the high-current regime ($Q\gg 1$) no longer depends on τ_{in}, there is a sizeable decrease in the quantum lifetime τ with increasing temperature,[12] which takes place because the electron-electron scattering contributes into τ. As a result, the Dingle factor decreases and the quantum contribution to the resistance becomes smaller as the electrons are heated. At a higher lattice temperature, when τ_{in} is smaller, the regime $Q\gg 1$ requires higher currents. The corresponding increase in heating[15] reduces the quantum contribution, so the amplitudes of inverted peaks never reach the theoretical limit and are expected to decrease with increasing lattice temperature. The slow suppression of the inverted peaks with further increase in the current (see the inset to Fig. 3) is explained by the same mechanism. This conclusion is supported by the experimental observation that the suppression is more efficient at lower magnetic fields, when the Dingle factor $\exp(-\pi/\omega_c\tau)$ is more sensitive to the temperature dependence of quantum lifetime τ. Theoretical calculations according to Eqs. (15) and (16), taking into account electron heating and the corresponding changes in τ and τ_{in}, demonstrate a reasonable agreement with the experiment for different samples and at different temperatures.

B. Origin of MIS peak splitting

The simple approach leading to Eqs. (15) and (16) fails to describe an interesting feature observed in our experiment: the current-induced splitting of the MIS oscillation peaks. This nonlinear behavior is well reproducible; we see it in different samples, and it was reported also by another group.[16] We have found that a possible explanation of this feature can be based on the theory presented in Sec. III, if higher harmonics of the distribution function δf_ε are taken into account. In order to demonstrate this, we have carried out a numerical solution of the system of equations (14) under some simplifying assumptions about the collision integral. In the first case, we have assumed equal probabilities for all possible electron-electron scattering processes, so the matrix $M_{jj',j_1j_1'}$ in Eq. (12) is replaced with a constant. Another limiting case is the complete neglect of intersubband transitions in electron-electron collisions, when $M_{jj',j_1j_1'} \propto \delta_{jj_1}\delta_{j'j_1'}$. Then, the coefficients γ_k and $C_{kk'}$ have been determined by using the density of states numerically calculated within the SCBA; see Eq. (6).

The results, corresponding to $I=120$ μA for sample B are presented in Fig. 6. In the low-field region, where the MIS peaks are inverted, the calculation shows a considerable increase in their amplitudes above 0.2 T, where the contribution of higher harmonics of the density of states becomes essential. This enhancement occurs because of the current-induced mixing between different harmonics of the distribution function, formally coming from the term with $\gamma_{k-k'}$ in the sum in Eq. (14). In contrast, in the linear regime the SCBA magnetoresistance appears to be close to the magnetoresistance calculated within the single-harmonic approximation [Eq. (16)]. Above 0.27 T, when the Landau levels become separated, one can see features associated with the specific semielliptic shape of the SCBA density of states. In the vicinity of the inversion field ($B_{inv}\simeq 0.4$ T) the contribution of the first harmonic of the distribution function is suppressed ($Q\simeq 1/3$) while the higher harmonics are still active. This leads to two sets of MIS peaks because higher harmonics of the density of states contain the factors $\cos(k\pi\Delta_{12}/\hbar\omega_c)$ describing higher harmonics of the MIS oscillations.

Above the inversion field, the resistance is considerably smaller than the resistance predicted by the single-harmonic approximation, and a splitting of the MIS peaks occurs. The splitting increases with the increase in the magnetic field. These effects are caused by the contribution of higher harmonics of the density of states in the collision integral. Indeed, in the single-harmonic approximation the collision integral contains only the outcoming term proportional to φ_ε.

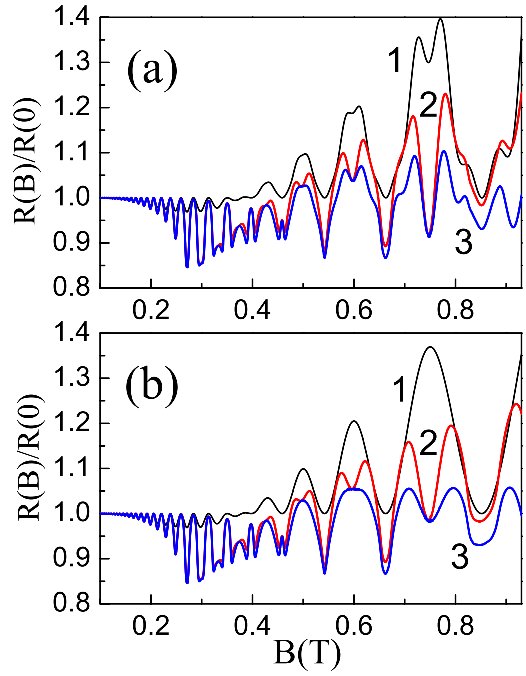

FIG. 6. (Color online) (a) Calculated magnetoresistance of sample B at $T=4.2$ K and $I=120$ μA. Plot 1 corresponds to simple theory [Eq. (16)], while the others represent the results of numerical solution of Eq. (14) for the cases of subband-independent electron-electron scattering (2) and only intrasubband electron-electron scattering (3). (b) The same plots, where the SdHO contribution is excluded. The density of states is found within the SCBA.

This approximation becomes insufficient in higher magnetic fields, when incoming terms in the collision integral (12) are also important, so the relaxation of the distribution function, which counteracts the diffusion of electrons in the energy space, becomes less efficient. Therefore, the effect of the current on the distribution function increases, and the resistance is lowered. The described suppression of the collision-integral term is more significant in the regions of the MIS resonances, when $\Delta_{12}/\hbar\omega$ is an integer, because the peaks of the density of states are the narrowest in these conditions, and the energies transferred in the electron-electron collisions, $\delta\varepsilon$, are small. Away from the MIS resonances, the energy space for electron-electron scattering increases, especially when the intersubband transitions are allowed (see plot 2 in Fig. 6). Therefore, the relaxation is less suppressed as compared to the center of the MIS peak, and the effect of the current is weaker. This consideration explains why the centers of the MIS peaks drop down, so the peak splitting takes place.

The SCBA has a limited applicability for the description of the density of electron states in the magnetic field. In particular, it leads to nonphysically sharp edges of the density of states, which generate the harmonics γ_k with large k in Eq. (14). This apparently leads to an overestimation of the effect of the current on the resistance in the region where the MIS peaks are inverted (see Fig. 6). To avoid such singularities, and to have a further insight into the problem of nonlinear magnetoresistance, we have considered the expression

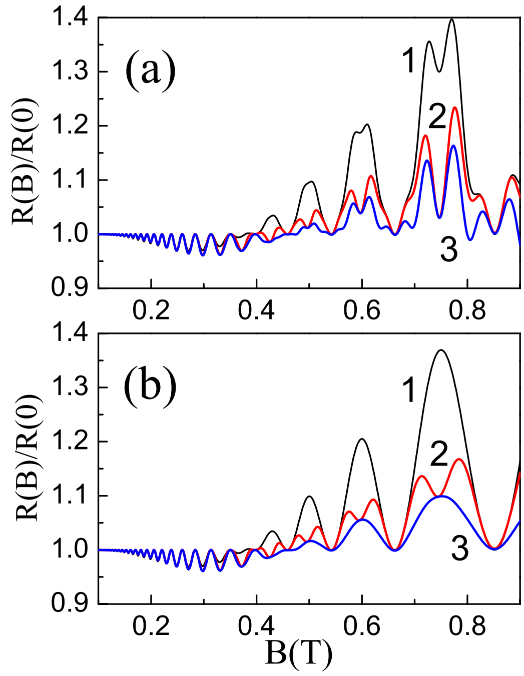

FIG. 7. (Color online) The same as in Fig. 6 for the Gaussian model of the density of states.

$$\mathcal{D}_{1,2\varepsilon}^{(G)} = \frac{\hbar\omega_c}{\sqrt{\pi}\Gamma(\omega_c)} \sum_{n=-\infty}^{\infty} \exp\left\{-\frac{[\varepsilon \pm \Delta_{12}/2 - \hbar\omega_c(n+1/2)]^2}{\Gamma^2(\omega_c)}\right\}, \tag{17}$$

which corresponds to the Gaussian model for the density of states and describes two independent sets of Landau-level peaks from each subband (strictly speaking, the Landau-level peaks are not independent because of elastic intersubband scattering, as follows from Eq. (5); see more details in Ref. 17). The magnetic-field dependence of the broadening energy Γ has been set to make the first [proportional to $\cos(2\pi\varepsilon/\hbar\omega_c)$] harmonics of $\mathcal{D}_{j\varepsilon}^{(G)}$ and $\mathcal{D}_{j\varepsilon}$ equal. The results of the calculations using $\mathcal{D}_{j\varepsilon}^{(G)}$ instead of the SCBA density of states are shown in Fig. 7. The magnetoresistance in the region of inversion appears to be nearly the same as predicted by the simple single-harmonic theory. In the region above the inversion field, the splitting of the MIS peaks does not take place if the intersubband electron-electron scattering is forbidden. This is understandable from the discussion given above: if different subbands contribute into the density of states independently, the efficiency of electron-electron collisions does not depend on the ratio $\Delta_{12}/\hbar\omega$ and the reduction in the collision integral owing to incoming terms causes just a uniform suppression of the whole MIS peak. In the SCBA, when the shape of $\mathcal{D}_{j\varepsilon}$ depends on this ratio, the splitting of the MIS peaks does not necessarily require the intersubband electron-electron scattering.

If the intersubband electron-electron scattering is allowed, the magnetoresistance pictures obtained within the Gaussian model, as well as within the SCBA model above the inversion point, qualitatively reproduce the features we observe experimentally in both our samples. For sample A, we have

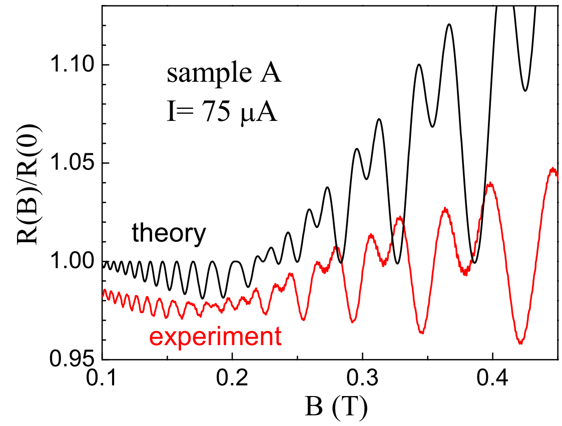

FIG. 8. (Color online) Comparison of the measured and the calculated nonlinear magnetoresistance in sample A at $T=4.2$ K and $I=75$ μA. The Gaussian model of the density of states and the assumption of subband-independent electron-electron scattering are used in the calculations.

put experimental and theoretical plots together in Fig. 8. Apart from a weak negative magnetoresistance at low fields and a slight decrease in the MIS oscillation frequency with increasing B (the features we see in all our samples[12,18] both in linear and nonlinear regimes), the agreement between experiment and theory is good.

V. CONCLUSIONS

The nonlinear behavior of magnetoresistance of 2D electron systems with increasing electric current is determined by such factors as Landau quantization, elastic and inelastic scatterings of electrons, and acceleration of electrons by the applied electric field. The theory,[8–10] which takes these factors into account, describes the observed resistance decrease[5,6] in terms of inversion of the quantum (i.e., associated with oscillating density of electron states) contribution to resistivity. The experimental studies[5,6] have verified different aspects of the theory, in particular, the role of inelastic-scattering time and quantum lifetime of electrons and the dependence of these characteristic times on the temperature and the magnetic field.

In two-subband systems such as DQWs, the quantum contribution to resistivity is modulated by the magnetointersubband (MIS) oscillations. This property opens ways for studying nonlinear response of 2D electron systems in magnetic fields. In our experiments, we have found that the current-induced inversion of the magnetoresistance shows up in DQWs as a flip of the MIS oscillation pattern, resembling the effect of the low-frequency microwave radiation.[18] To support this observation, we have developed a theory of nonlinear magnetoresistance in two-subband systems. Under reasonable approximations, the magnetoresistance is described by a simple expression (16). The theory suggests that the magnetic field B_{inv} corresponding to the inversion of the quantum contribution linearly depends on the current and is proportional to the square root of the inelastic-scattering time τ_{in}. Since in DQWs we determine B_{inv} directly, by the position of the flip point in the magnetoresistance, we have verified these theoretical predictions in detail. The inelastic-scattering time and its temperature dependence, found in this way, are in agreement with theoretical estimates. Another prediction of the theory, the saturation of the inverted quantum contribution to resistivity at high current density, is also verified. In the saturation regime, the decrease in the quantum lifetime of electrons due to heating leads to a decrease in the inverted MIS peaks amplitudes with increasing current.

Apart from this, we have observed a splitting of the MIS oscillation peaks, which is sensitive to the current and occurs at the fields above the inversion point, where the quantum contribution to resistivity is positive. This phenomenon has no analog in single-subband 2D systems. It cannot be explained within the approach accounting only for the first oscillatory harmonic of the nonequilibrium distribution function and leading to magnetoresistance (16). We have given a theoretical explanation of the peak splitting based on the modification of inelastic scattering owing to the Landau quantization. The key point of our theory is a suppression of inelastic relaxation due to narrowing of energy space for electron-electron scattering in the region of MIS resonances. The quantitative description of this effect required a detailed numerical analysis including consideration of higher harmonics of both the density of states and the distribution function.

In summary, we have carried out both experimental and theoretical studies of nonlinear magnetoresistance in two-subband electron systems formed in double quantum wells and obtained a good agreement between experimental and theoretical results. The basic principles of the nonlinear transport theory developed previously for single-subband systems can be applied to the case of two subbands. However, the understanding of the phenomena observed in double quantum wells has required consideration of specific features of the density of states and scattering mechanisms in these systems.

ACKNOWLEDGMENT

This work was supported by CNPq and FAPESP (Brazilian agencies).

[1] G. Eber, K. von Klitzing, K. Ploog, and G. Weimann, J. Phys. C **16**, 5441 (1983); M. E. Cage, R. F. Dziuba, B. F. Field, E. R. Williams, S. M. Girvin, A. C. Gossard, D. C. Tsui, and R. J. Wagner, Phys. Rev. Lett. **51**, 1374 (1983).

[2] C. L. Yang, J. Zhang, R. R. Du, J. A. Simmons, and J. L. Reno, Phys. Rev. Lett. **89**, 076801 (2002).

[3] W. Zhang, H. S. Chiang, M. A. Zudov, L. N. Pfeiffer, and K. W. West, Phys. Rev. B **75**, 041304(R) (2007).

[4] A. A. Bykov, J. Q. Zhang, S. Vitkalov, A. K. Kalagin, and A. K. Bakarov, Phys. Rev. B **72**, 245307 (2005).

[5] J.-Q. Zhang, S. Vitkalov, A. A. Bykov, A. K. Kalagin, and A. K. Bakarov, Phys. Rev. B **75**, 081305(R) (2007).

[6] J. Q. Zhang, S. Vitkalov, and A. A. Bykov, Phys. Rev. B **80**, 045310 (2009).

[7] X. L. Lei, Appl. Phys. Lett. **90**, 132119 (2007).

[8] I. A. Dmitriev, M. G. Vavilov, I. L. Aleiner, A. D. Mirlin, and D. G. Polyakov, Phys. Rev. B **71**, 115316 (2005).

[9] M. G. Vavilov, I. L. Aleiner, and L. I. Glazman, Phys. Rev. B **76**, 115331 (2007).

[10] M. Khodas and M. G. Vavilov, Phys. Rev. B **78**, 245319 (2008).

[11] M. G. Vavilov and I. L. Aleiner, Phys. Rev. B **69**, 035303 (2004).

[12] N. C. Mamani, G. M. Gusev, T. E. Lamas, A. K. Bakarov, and O. E. Raichev, Phys. Rev. B **77**, 205327 (2008).

[13] See, for example, O. E. Raichev and F. T. Vasko, Phys. Rev. B **74**, 075309 (2006).

[14] Y. Ma, R. Fletcher, E. Zaremba, M. D'Iorio, C. T. Foxon, and J. J. Harris, Phys. Rev. B **43**, 9033 (1991).

[15] According to our calculations, application of the current of 400 μA to sample B at 4.2 K increases electron temperature to 6.3 K.

[16] A. A. Bykov, JETP Lett. **88**, 64 (2008).

[17] O. E. Raichev, Phys. Rev. B **59**, 3015 (1999).

[18] S. Wiedmann, G. M. Gusev, O. E. Raichev, T. E. Lamas, A. K. Bakarov, and J. C. Portal, Phys. Rev. B **78**, 121301(R) (2008).